International Journal of Theoretical Physics, Vol. 31, No. 1, 1992

Higgs Field and a New Scalar–Tensor Theory of Gravity

H. Dehnen,[1] H. Frommert,[1] and F. Ghaboussi[1]

Received January 18, 1991

The combination of Brans and Dicke's idea of a variable gravitational constant with the Higgs-field mechanism of elementary particle physics results in a new theory of gravity. Einstein's theory is realized after symmetry breaking in the neighborhood of the Higgs-field ground state.

There exist today in the literature two fundamental scalar fields connected with the mass problem. First of all Brans and Dicke (1961; see also Jordan, 1955) introduced a scalar field with the intention of following Mach's principle (Einstein, 1917, 1973) that the active as well as passive gravitational mass $m_0\sqrt{G}$, which means the gravitational "constant" G, is not a constant but a function determined by the other particles of the universe. In this way also the problem of Dirac's large cosmological numbers should be solved. Second, in elementary particle physics the inertial mass m_0 is generated in a gauge-invariant theory by the interaction with the scalar Higgs field, the source of which is also given by the particles in the universe (Dehnen *et al.*, 1990). Because of the identity of gravitational and inertial mass (equivalence principle), it seems to be meaningful, if not even necessary, to identify these two approaches. Then the Lagrangian density has the proposed form ($\hbar=1, c=1$)

$$\mathscr{L}=\left[\frac{1}{16\pi}\alpha\phi^\dagger\phi R+\tfrac{1}{2}\phi^\dagger_{\|\mu}\phi^{\|\mu}-V(\phi)\right]\sqrt{-g}+L_M\sqrt{-g} \tag{1}$$

with the Higgs potential (μ^2, λ real-valued constants)

$$V(\phi)=\frac{\mu^2}{2}\phi^\dagger\phi+\frac{\lambda}{4!}(\phi^\dagger\phi)^2+\frac{3}{2}\frac{\mu^4}{\lambda} \tag{1a}$$

[1]Universität Konstanz, Postfach 55 60, W-7750 Konstanz, Germany.

0020-7748/92/0100-0109$06.50/0

Here ϕ is a $U(N)$ isovector, $\|\mu$ means its covariant derivative, R is the Ricci scalar, and α is a dimensionless factor, whereas L_M contains the fermionic and massless bosonic fields belonging to the inner gauge group $U(N)$. Obviously, the positive Higgs-field quantity $\phi^\dagger\phi$ [cf. equation (9a)] plays the role of a variable reciprocal gravitational "constant." Formula (1) is related to a generalization of Brans and Dicke's theory proposed by Bergmann (1968) and Wagoner (1970) as well as by Zee (1979, 1980) [see also references in Zee (1980)]. We want to point here to some interesting features of the ansatz (1), which unifies gravity with the other interactions using a minimum of effort.

Before symmetry breaking the theory following from (1) contains no gravitational constant; the only dimensional parameters are those of the Higgs potential. Such a theory of gravity may be renormalizable according to the criterion given by de Witt (1979), although it is not unitary. Concerning symmetry breaking, the ground state of the Higgs field is given by ($\mu^2<0$)

$$\phi_0^\dagger\phi_0 = v^2 = \frac{-6\mu^2}{\lambda} \tag{2}$$

with

$$V(\phi_0) = 0 \tag{2a}$$

By this ground state the quantity

$$G = (\alpha v^2)^{-1} \tag{3}$$

related to Newton's gravitational constant (see below), as well as the mass of the gauge bosons

$$M_W = \sqrt{\pi}\, g v \tag{4}$$

are determined [g is the coupling constant of the gauge group $U(N)$]. Accordingly the factor α denotes the ratio

$$\alpha \simeq (M_P/M_W)^2 \gg 1 \tag{5}$$

where M_P is the Planck mass.

The field equations for gravity and the Higgs field following from (1) take the form

$$\begin{aligned} R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu} &+ \frac{8\pi}{\alpha\phi^\dagger\phi} V(\phi) g_{\mu\nu} \\ &= -\frac{8\pi}{\alpha\phi^\dagger\phi} T_{\mu\nu} - \frac{4\pi}{\alpha\phi^\dagger\phi}[\phi^\dagger_{\|\mu}\phi_{\|\nu} + \phi^\dagger_{\|\nu}\phi_{\|\mu}] \\ &\quad + \frac{4\pi}{\alpha\phi^\dagger\phi}\phi^\dagger_{\|\lambda}\phi^{\|\lambda} g_{\mu\nu} - \frac{1}{\phi^\dagger\phi}[(\phi^\dagger\phi)_{\|\mu\|\nu} - (\phi^\dagger\phi)^{\|\beta}{}_{\|\beta} g_{\mu\nu}] \end{aligned} \tag{6}$$

and

$$\phi^{\|\mu}{}_{\|\mu} - \frac{1}{8\pi}\alpha\phi R + \mu^2\phi + \frac{\lambda}{6}(\phi^\dagger\phi)\phi = 0 \tag{7}$$

as well as the adjoint equation of (7). Here $T_{\mu\nu}$ is the symmetric energy-momentum tensor belonging to $L_M\sqrt{-g}$ in (1) alone. The conservation laws of energy and momentum read

$$T_\mu{}^\nu{}_{\|\nu} = 0 \tag{8}$$

Now we perform the symmetry breaking and introduce the unitary gauge. If with respect to (2)

$$\phi_0 = vN; \qquad N^\dagger N = 1; \qquad N = \text{const} \tag{9}$$

represents the ground state, the Higgs field ϕ can be brought within the unitary gauge into the form:

$$\phi = \rho N, \qquad \rho^2 = \phi^\dagger\phi \tag{9a}$$

For the following we use therefore instead of ϕ the real-valued field quantity

$$\varphi = \rho/v \tag{10}$$

($\varphi = 1$ represents the ground state). Restricting ourselves to the field equations for gravity, i.e., the metric $g_{\mu\nu}$ and the Higgs field φ, we find from (6) and (7) ($|_\mu$ means the usual partial derivative)

$$\begin{aligned} R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} &+ \frac{12\pi}{\alpha v^2}\frac{\mu^4}{\lambda}\varphi^{-2}(\varphi^2-1)^2 g_{\mu\nu} \\ &= -\frac{8\pi}{\alpha v^2}\varphi^{-2}T_{\mu\nu} - \frac{8\pi}{\alpha}\left(1+\frac{\alpha}{4\pi}\right)\varphi^{-2}\varphi_{|\mu}\varphi_{|\nu} \\ &+ \frac{4\pi}{\alpha}\left(1+\frac{\alpha}{2\pi}\right)\varphi^{-2}\varphi^{|\lambda}\varphi_{|\lambda}g_{\mu\nu} - 2\varphi^{-1}[\varphi_{|\mu\|\nu} - \varphi^{|\lambda}{}_{\|\lambda}g_{\mu\nu}] \end{aligned} \tag{11}$$

and

$$\frac{4\pi}{\alpha}\left(1+\frac{3\alpha}{4\pi}\right)\varphi^{2|\mu}{}_{\|\mu} + \frac{48\pi}{\alpha v^2}\frac{\mu^4}{\lambda}(\varphi^2-1) = \frac{8\pi}{\alpha v^2}T \tag{12}$$

With respect to (3) and (5) we obtain from (11) and (12) the final result:

$$R_{\mu\nu}-\tfrac{1}{2}Rg_{\mu\nu}+12\pi G\frac{\mu^4}{\lambda}\varphi^{-2}(\varphi^2-1)^2g_{\mu\nu}$$
$$=-8\pi G\varphi^{-2}T_{\mu\nu}-2\varphi^{-2}\varphi_{|\mu}\varphi_{|\nu}$$
$$+2\varphi^{-2}\varphi^{|\lambda}\varphi_{|\lambda}g_{\mu\nu}-2\varphi^{-1}[\varphi_{|\mu\|\nu}-\varphi^{|\lambda}{}_{\|\lambda}g_{\mu\nu}] \quad (13)$$

and

$$\varphi^{2|\mu}{}_{\|\mu}+16\pi G\frac{\mu^4}{\lambda}(\varphi^2-1)=\frac{8\pi G}{3}T \quad (14)$$

There are two very important differences with respect to Brans and Dicke's scalar–tensor theory. First, the scalar field φ possesses a finite range $l=M_\varphi^{-1}$ corresponding to the mass term in (14) according to which the excited Higgs field has the mass square

$$M_\varphi^2=16\pi G\frac{\mu^4}{\lambda} \quad (15)$$

This is smaller than the usual value by the factor α^{-1}. In this connection we note that G in (3), (13), and (14) represents Newton's gravitational constant only up to a factor of the order of one. The exact connection between G and the Newtonian value G_N is given by the Newtonian limit of (13) and (14) and this depends, as shown below, on the value of l for the range of the scalar field. In case of a suitable choice of this range, also no difficulties with respect to the solar-relativistic effects or gravitational waves are to be expected; however, the possibility of a fifth force of Yukawa type is given.

Second, there exists, according to the left-hand side of (13), a cosmological function (instead of a cosmological constant)

$$\lambda(\varphi)=12\pi G\frac{\mu^4}{\lambda}\varphi^{-2}(\varphi^2-1)^2 \quad (16)$$

which is necessarily positive. This is very interesting because a positive value of a cosmological function (constant) corresponds to a positive mass density and a negative pressure, so that this theory could solve the problem of missing mass and of inflation in cosmology automatically.

For the ground state of the Higgs field ($\varphi\equiv 1$) the cosmological function $\lambda(\varphi)$ vanishes [see also (2a)] and from (13) and (14) it follows that

$$R_{\mu\nu}-\tfrac{1}{2}Rg_{\mu\nu}=-8\pi GT_{\mu\nu}, \qquad T=0 \quad (17)$$

This is Einstein's theory with lightlike matter. Einstein's theory is realized only after symmetry breaking in the neighborhood of the ground state. Of course, in case of vanishing energy-momentum tensor the ground state is realized by the Minkowski space-time and $\varphi=1$.

Finally we investigate the Newtonian limit. For this we set

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}; \qquad \varphi=1+\zeta \tag{18}$$

and linearize with respect to $|h_{\mu\nu}|\ll 1$ and $|\zeta|\ll 1$ [$\eta_{\mu\nu}=\mathrm{diag}(1, -1, -1, -1)$]. In this way we obtain from (13) and (14) using the de Donder gauge $h_{\mu}{}^{\nu}{}_{|\nu}-\frac{1}{2}h_{|\mu}=0$:

$$h_{\mu\nu}{}^{|\lambda}{}_{|\lambda}=-16\pi G(T_{\mu\nu}-\tfrac{1}{3}T\eta_{\mu\nu})+32\pi G\frac{\mu^4}{\lambda}\zeta\eta_{\mu\nu}-4\zeta_{|\mu|\nu} \tag{19}$$

and

$$\zeta^{|\lambda}{}_{|\lambda}+16\pi G\frac{\mu^4}{\lambda}\zeta=\frac{4\pi G}{3}T \tag{20}$$

Because of the geodesic equation of motion of a free point particle in consequence of (8)

$$h_{00}=2\phi_{\mathrm{N}} \tag{21}$$

is valid, where ϕ_{N} is the Newtonian gravitational potential. Insertion of (21) into (19) yields

$$\phi_{\mathrm{N}}{}^{|\lambda}{}_{|\lambda}=-8\pi G(T_{00}-\tfrac{1}{3}T)+16\pi G\frac{\mu^4}{\lambda}\zeta-2\zeta_{|0|0} \tag{22}$$

For a point particle of mass M at rest in the origin the solution of (20) reads

$$\zeta=\frac{MG}{3r}e^{-r/l}, \qquad l^2=\frac{\lambda}{16\pi G\mu^4} \tag{23}$$

Herewith the solution of (22) for a point particle takes the form

$$\phi_{\mathrm{N}}=-\frac{MG}{r}(1+\tfrac{1}{3}e^{-r/l}) \tag{24}$$

Consequently $G=G_\infty$ is valid, where G_∞ is the Newtonian gravitational constant G_{N} determined by a torsion-balance experiment in the laboratory in the case $r\gg l$. In case of $r\ll l$ one finds $G=\frac{3}{4}G_0$ with $G_0=G_{\mathrm{N}}$. It is interesting that such gravitational potentials, where the usual r^{-1} potential is supplemented by a Yukawa term, are discussed in connection with the fifth force (Fischbach *et al.*, 1986; Eckhardt *et al.*, 1988) and in view of the flat rotation curves of spiral galaxies (Sanders, 1986).

In the static linear Newtonian limit, the potential equations following from (20) and (22) are

$$\Delta\zeta - \frac{l}{l^2}\zeta = -\frac{4\pi G}{3}\rho, \qquad \Delta\phi_{\mathrm{N}} + \frac{1}{l^2}\zeta = \frac{16\pi G}{3}\rho \tag{25}$$

instead of the Poisson equation. The meaning of ζ is that $\varphi^{-2} = 1 - 2\zeta$ represents the variability of the gravitational "constant" in first order [cf. equation (13)]; it decreases in view of (23) with decreasing distance from a mass. Therefore the gravitational action of massive objects will be damped. The cosmological function (16) is of second order in ζ and therefore not yet contained in (25); however, its absolute value increases with decreasing distance from a mass. Finally we note that the scalar field ζ acts in the potential equation (25) for ϕ_{N} as a negative mass density (antigravity) (cf. Sanders, 1986). The applications of these ideas to modern astrophysical and cosmological questions are in preparation.

REFERENCES

Bergmann, P. G. (1968). *International Journal of Theoretical Physics*, **1**, 25.

Brans, C., and Dicke, R. (1961). *Physical Review*, **124**, 925.

Dehnen, H., *et al.* (1990). *International Journal of Theoretical Physics*, **29**, 537.

De Witt, B. S. (1979). The formal structure of quantum gravity, in *Recent Developments in Gravitation* (M. Levy and S. Deser, eds.), Plenum Press, New York, p. 300.

Eckhardt, D. H., *et al.* (1988). *Physical Review Letters*, **60**, 2567.

Einstein, A. (1917). *Sitzungsberichte der Preussischen Akademie der Wissenschaften Berlin*, p. 142, § 2.

Einstein, A. (1973). *Grundzüge der Relativitätstheorie*, F. Vieweg, Braunschweig, p. 98.

Fischbach, E., *et al.* (1986). *Physical Review Letters*, **56**, 3.

Jordan, P. (1955). *Schwerkraft und Weltall*, F. Vieweg, Braunschweig.

Sanders, R. H. (1986). *Astronomy and Astrophysics*, **154**, 135.

Wagoner, R. V. (1970). *Physical Review D*, **1**, 3209.

Zee, A. (1979). *Physical Review Letters*, **42**, 417.

Zee, A. (1980). *Physical Review Letters*, **44**, 703.